**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. *0.2676*
4. 0.5
5. 0.6987

Time taken to service a car = 50 minutes.

P (X > 50) = 1 - P (X ≤ 50).

Z = (X - 45)/8.0

By using the normal table to find

P(X ≤ 50) = P(Z ≤ (50 - 45)/8.0) = P(Z ≤ 0.625)=73.4%

Probability that the service manager will not meet his demand will be

🡺 100-73.4 🡺 26.6%

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**FALSE.**

When the employees are older than 44, this will shift the*μ*  towards 44 with considering std. Which will contradict with the given data.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

TRUE.

1. If *X1* ~ *N* (μ, σ2) and *X*2 ~ *N* (μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

*X*~ *N* (μ1, σ1^2) and *Y*~ *N*(μ2, σ2^2) are two independent random variables then X+Y~N(μ1+ μ2, σ1^2 + σ2^2) and X-Y~N(μ1- μ2, σ1^2 + σ2^2)

Similarly if Z = aX + bY, where X and Y are as defined above, i.e. Z is

linear combination of X and Y, then Z ∼ N(aµ1 + bµ2, a^2σ1^2 + b^2σ2^2 ).

Therefore, in the question 2X1~ N (2 µ,4 σ^2) and

X1+X2 ~ N(µ + µ, σ^2 + σ^2 ) ~ N(2 µ, 2σ^2 )

2X1-(X1+X2) = N (4µ,6 σ^2)

1. Let X ~ N (100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. *48.5, 151.5*
6. 90.1, 109.9

norm.ppf (0.005,100,20)

48.483413929021985

norm.ppf (0.995,100,20)

151.516586070978

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

print ('Upper Limit:',norm.ppf(0.025,45\*5,3))

print ('Lower Limit:',norm.ppf(0.975,45\*5,3))

print ('Upper Limit:',norm.ppf(0.025,45\*7,3))

print('Lower Limit :',norm.ppf(0.975,45\*7,3))

*Upper Limit: 219.12010804637984*

*Lower Limit: 230.87989195362016*

*Upper Limit: 309.1201080463798*

*Lower Limit: 320.8798919536202*

print ('The Rupee will be in the range of [',219.12010804637984+309.1201080463798,',',230.87989195362016+320.8798919536202,']')

*The Rupee will be in the range of [ 528.2402160927596, 551.7597839072404]*

1. Specify the 5th percentile of profit (in Rupees) for the company

print(norm.ppf(0.05,45\*7,3))

print(norm.ppf(0.05,45\*5,3))

*310.0654391191456*

*220.0654391191456*

print ('5th percentile of profit (in Rupees)

=', 310.0654391191456+220.0654391191456)

*5th percentile of profit (in Rupees) = 530.1308782382912*

1. Which of the two divisions has a larger probability of making a loss each year?

2nd Division